Pre-calculus

Number and Operations

Enduring Understandings:

Every point on a line corresponds to a Real Number.

Properties of the Real Number System may or may not hold in other mathematical systems.

The complex number system consists of all the real and imaginary numbers.

Solutions to problems call for estimates, approximations to an appropriate degree of precision or exact answers.

Mental computations are the basis for making reasonable estimates and sensible predictions.

Some problem-solving strategies and techniques may create extraneous solutions.

Essential Questions:

What does it mean to be "computationally fluent"?

1. What is a mathematical system?
2. How do you know when an estimate, approximation or exact answer is appropriate?
3. How are mental calculations useful?

Outcomes:

The student will:

A. Understand different ways to represent and relate numbers and number systems.

1. Develop a deeper understanding and flexibility with rational and real numbers
2. Develop a deeper understanding of complex number systems and operations with complex numbers
3. Understand the usefulness of complex numbers in application.
4. Distinguish when an exact answer versus an approximation is appropriate

B. Understand how to compute fluently and make reasonable estimates.

1. Enhance fluency in operations with rational and irrational numbers (Real Numbers)
2. Enhance fluency in operations with complex numbers.
3. Focus on appropriate use of technology to solve problems
4. Judge the reasonableness of numerical results, whether real, complex, or trigonometric.
5. Understand that often extraneous solutions are introduced by methodology and be able to eliminate these as viable solutions
6. Select an appropriate degree of accuracy for numerical results.
7. Decide whether a solution is appropriate based on the context of the situation, and eliminate inappropriate solutions

Algebra

Enduring Understandings:

Modeling involves identifying and selecting features of a real-world situation, representing those features symbolically, analyzing and reasoning about the model and the characteristics of the situation, and considering the accuracy and limitations of the model.

Linear, exponential, logarithmic, polynomial, rational, periodic, step and absolute value functions and their inverses (if they exist) can be represented verbally, numerically, graphically, and analytically to understand patterns and relationships.

Algebraic properties govern the fluent manipulation of symbols in expressions, equations, and inequalities.

Rates of change can be represented mathematically.

Essential Questions:

1. How do we analyze and understand patterns, relations and functions?
2. How can verbal, numerical, graphical and analytical representations be used to analyze and solve problems?
3. What does it mean to be “symbolically fluent”?

Outcomes:

The student will:

A. Understand patterns, relations and functions

1. Using generalized patterns, create explicitly defined and recursively defined models, including appropriate notation.
2. Represent relations and function numerically, analytically, verbally and graphically.
3. Analyze functions by:
    a. Finding, recognizing and applying rates of change(constant and not constant).
    b. Finding, recognizing and applying roots, intercepts, asymptotes, local and global behavior.
4. Compare and contrast families of functions; focusing on exponential, logarithmic, polynomial, periodic, step and absolute value; including their inverses (if they exist).

B. Understand how to use algebraic symbols to represent and analyze mathematical situations and structures.
1. Explore the meaning of equivalent forms of expressions, equations, inequalities and relations.
2. Recall and use the properties of algebra in real and complex number systems
    a. Apply the commutative, associative and distributive properties
    b. Apply the properties of equality
    c. Apply properties of exponents and radicals
    d. Apply composite functions in analysis of inverses
3. Write equivalent forms of equations, inequalities and systems of equations
4. Use multiple methods to fluently solve equations, inequalities and systems of equations mentally, with paper and pencil and with technology when appropriate

C. Understand how contextual situations can be modeled quantitatively.
1. Construct suitable mathematical models to represent a situations focusing on ALL families of functions
2. Draw conclusions about a situation using a mathematical model

Geometry

Enduring Understandings:

Representation of geometric ideas within pre-calculus and the relationships formed allows multiple approaches to geometric problems and connects geometric interpretations to other contexts.

Essential Questions:

1. How does geometric visualization enhance the study of pre-calculus?

2. How does geometry explain or describe the structure of algebra?

Outcomes:

The student will:

A. Understand and explore properties of right (and non-right) triangles to determine lengths and angle measures
    1. Use the Pythagorean theorem
    2. Use basic trigonometric ratios (sine, cosine, tangent)
    3. Use the inverse trigonometric ratios
    4. Use the Law of Sines and the Law of Cosines

B. Understand the components and geometric relationships on the Cartesian Plane.
    1. Calculate distance, midpoint, slope and slope relationships of figures
    2. Relate geometry and algebra by connecting graphical representations to their algebraic forms
        a. Recognize graphs of functions and relations
        b. Recognize conic sections and their models
        c. Utilize vectors as geometric representations in context

C. Understand transformations in mathematical situations
    1. Represent and apply translations, reflections, rotations and dilations of functions and relations on a coordinate axis.

Measurement

Enduring Understandings:

Use of calculation and computer technologies for gathering and displaying data requires making strategic choices for selection of scale and viewing window.

Reasonable estimates and sensible judgment about the precision and accuracy of measurement values is important.

Essential Questions:

1. How are accuracy and precision in measurement determined?
2. How can measurement tools be utilized most effectively and efficiently?

Outcomes:

The student will:

A. Understand measurable attributes of objects and units, systems and processes of measurement

1. Use appropriate units and scales when graphing on the Cartesian plane
2. Understand the meaning of radian measures and the radian measurement system
3. Connect measurements in radians and degrees, converting from one unit to the other in a fluent manner

B. Use appropriate techniques and formulas to determine measurements

1. Analyze precision, accuracy and approximate error in measurement situations
2. Convert between units of measurement fluently

Data Analysis and Probability

(not addressed in Pre-calc)

Problem Solving

Enduring Understandings:

Problem solving requires:

- knowledge of mathematical content
- knowledge of problem-solving strategies
- effective self-monitoring and
- a disposition for posing and solving problems.

Essential Questions:

What do you do when you don't know what to do?

1. How do you identify strategies to use when solving problems?
2. How do you know you have solved the problem?

Outcomes:

The student will:

A. Understand that there are multiple ways to solve a problem
    1. Employ appropriate method(s) for solving a problem
    2. Apply a variety of appropriate strategies
    3. Adapt a previous strategy to a new problem-solving situation

B. Understand how to monitor problem solving
    1. Recognize when a problem solving strategy is not working
    2. Compare and contrast strategy efficiency
    3. Determine when the problem has been solved and the solution is reasonable.

C. Understand that they can pose their own problems
    1. Formulate interesting extensions to a problem
    2. Propose problems to be solved

Reasoning and Proof

Enduring Understandings:

Proof is a way to explain, justify, and communicate mathematics.

Informal observations lead to specific examples that are generalized and then proven.

Essential Questions:

1. How do we develop mathematical conjectures?
2. How do you construct a convincing argument?
3. How do you know you have a convincing argument?

Outcomes:

The student will:

A. Understand mathematical conjectures
    1. Make mathematical conjectures
    2. Investigate mathematical; conjectures that have already been proposed

B. Understand mathematical arguments and proof
    1. Evaluate written proofs
    2. Develop mathematical arguments and proof

C. Understand multiple methods of proof

1. Utilize deductive reasoning, indirect proof and inductive reasoning to defend and justify one's ideas

Communication

Enduring Understandings:

Quality mathematical thinkers develop clear and coherent communication using appropriate mathematical language, notation and representation while considering their audience.

Writing is a valuable way of reflecting on and solidifying what one knows.

Essential Questions:

What is the language of mathematics?
Why do I need to explain my thinking when I solve problems?

1. How do you "show what you know" mathematically?

Outcomes:

The student will:

A. Understand the language of mathematics
   1. Convey mathematical ideas orally
   2. Convey mathematical ideas in written form

B. Understand that mathematics is a language unto itself
   1. Utilize mathematical language and symbols to communicate correctly and coherently to peers, teachers and others

Representation

Enduring Understandings

Mathematics is the "science of patterns" and representations are the means by which those patterns are recorded and analyzed.

Mathematical representations ***illustrate*** essential features of a situation and clarify mathematical relationships.

Essential Questions

1. Why is mathematics considered the "science of patterns"?
2. How do we use mathematical representations to illustrate patterns?
3. How do mathematical representations model situations?

Outcomes:

The student will:

A. Understand that mathematical representations facilitate communication of ideas

1. Create multiple representations that organize, record and communicate mathematical ideas
2. Use pattern recognition to generalize mathematical ideas

B. Understand that there are common mathematical structures across different contexts

1. Select, apply and translate among mathematical representations to solve problem
2. Use representations to model and interpret, physical, social and mathematical phenomena

Connections

Enduring Understandings:

Linking mathematical ideas, within mathematics and across content areas, develops robust understanding of problems.

Essential Questions:

1. What is the big picture in mathematics?
2. Why does mathematics matter to society?

Outcomes:

The student will:

A. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole

1. Recognize and use connections among mathematical ideas
2. Develop multiple approaches to the same problem leading to equivalent results
3. Use insights gained in one context to prove or disprove conjectures generated in another context

B. Understand that mathematics exists in contexts outside of the mathematics classroom

1. Connect mathematics problems to real world situations
2. Apply mathematical thinking in problems they encounter outside of the mathematics classroom